Formulas

$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad (x \neq -1) \qquad \int \frac{1}{x} dx = \ln|x| + C \qquad \int e^x dx = e^x + C$

Group 1 :

1. $\int 2x^4 + e^x - \frac{1}{2x} dx = 2\int x^4 dx + \int e^x dx - \frac{1}{2}\int \frac{1}{x} dx$

$= 2\frac{x^5}{5} + e^x - \frac{1}{2}\ln|x| + C$

2. $\int \frac{5}{x^2} + \frac{8}{x^5} dx = \int 5x^{-2} + 8x^{-5} dx = 5\frac{x^{-1}}{-1} + 8\frac{x^{-4}}{-4} + C$

3. $\int \frac{5}{x^3} - \frac{1}{3x} + 4\sqrt{x^3} - 8 dx = \int 5x^{-3} - \frac{1}{3}\frac{1}{x} + 4x^{\frac{3}{2}} - 8 dx$

$= 5\frac{x^{-2}}{-2} - \frac{1}{3}\ln|x| + 4\frac{x^{\frac{5}{2}}}{\frac{5}{2}} - 8x + C$

4. $\int (x^2-2)(x^2+1) dx$ key: $\int f(x)g(x) dx \neq \int f(x)dx \cdot \int g(x)dx$

$= \int x^4 - x^2 - 2 dx = \frac{x^5}{5} - \frac{x^3}{3} - 2x + C$

5. $\int u^2(\sqrt{u} + \sqrt[4]{u}) du$

$= \int u^2(u^{\frac{1}{2}} + u^{\frac{1}{4}}) du = \int u^{2.5} + u^{2.25} du = \frac{u^{3.5}}{3.5} + \frac{u^{3.25}}{3.25} + C$

6. $\int \frac{y^2 - 3y^4}{y^2} dy$

$= \int 1 - 3y^2 dy = y - 3\frac{y^3}{3} + C = y - y^3 + C$

7. $\int \frac{4+u^2}{-8u^3} du = -\frac{1}{8}\int \frac{4+u^2}{u^3}\frac{}{u^3} du$

$= -\frac{1}{8}\int 4u^{-3} + \frac{1}{u} du = -\frac{1}{8}\left(4\frac{u^{-2}}{-2} + \ln|u|\right) + C$

Group 2

1. $\int_1^y e^x + \frac{1}{x}\,dx = e^x + \ln|x| \Big|_{x=1}^{x=y} = e^y + \ln|y| - (e^1 + \ln 1)$

$= e^y + \ln|y| - e.$

2. ~~$\int_1^2 \left(\frac{d}{dx}\sqrt{1+t^2}\right) dx$~~ (Typo: everything should be in "x". No "t" there.)

$\int_1^2 \left(\frac{d}{dx}\sqrt{1+x^2}\right) dx = \sqrt{1+x^2}\Big|_{x=1}^{x=2} = \sqrt{5} - \sqrt{3}$

3. $F(x) = e^{ax}$

(a) $F'(x) = e^{ax} \cdot a$

(b) By (a), we have $\int a e^{ax}\,dx = e^{ax} + C$

Hence, $\int e^{ax}\,dx = \frac{e^{ax}}{a} + C$

4. $\int_{-2}^{2} f(x)\,dx = \int_{-2}^{1} x\,dx + \int_1^2 \frac{1}{x}\,dx$

$= \frac{x^2}{2}\Big|_{x=-2}^{x=1} + \ln|x|\Big|_{x=1}^{x=2}$

$= \frac{1}{2} - 2 + \ln 2 - \ln 1 = \ln 2 - \frac{3}{2}$